Computer Science 3 - 2010

Programming Language Translation

Practical for Week 22, beginning 20 September 2010 - Solutions

This tutorial/practical was not always well done. Many people could "guess" the answers, but could not or did not justify their conclusions. If set in exams, in these sorts of questions it is important to do so.

As usual, you can find a "solution kit" as PRAC22A.ZIP or PRAC22AC.ZIP if you wish to experiment further.

Task 1 - Meet the family

Consider the following grammar:

```
COMPILER Home

IGNORE CHR(O) .. CHR(31)

PRODUCTIONS

Home = Family { Pets } [ Vehicle ] "house" .

Pets = "dog" [ "cat" ] | "cat" .

Vehicle = ( "scooter" | "bicycle" ) "fourbyfour" .

Family = Parents { Children } .

Parents = [ "Dad" ] [ "Mom" ] | "Mom" "Dad" .

Child = "Helen" | "Margaret" | "Alice" | "Robyn" | "Cathy"

| "Janet" | "Anne" | "Ntombizodwa" | "Ntombizanele" .

END Home.
```

Analyse this grammar in detail.

The first point to be made is that this is not a reduced grammar. The non-terminal Child is unreachable, and there is no way that the non-terminal Children can be derived to anything, let alone to terminals. Presumably what was meant was

```
COMPILER Home

IGNORE CHR(0) .. CHR(31)

PRODUCTIONS

Home = Family { Pets } [ Vehicle ] "house" .

Pets = "dog" [ "cat" ] | "cat" .

Vehicle = ( "scooter" | "bicycle" ) "fourbyfour" .

Family = Parents { Child } .

Parents = [ "Dad" ] [ "Mom" ] | "Mom" "Dad" .

Child = "Helen" | "Margaret" | "Alice" | "Robyn" | "Cathy"

| "Janet" | "Anne" | "Ntombizodwa" | "Ntombizanele" .
```

```
END Home.
```

If we introduce extra non-terminals to eliminate the [] and {} metabrackets we might get:

```
COMPILER Home
IGNORE CHR(0) .. CHR(31)
PRODUCTIONS
              = Family AllPets Vehicle "house" .
= Pets AllPets |
  Home
  AllPets
  Pets
              = "dog" OptionalCat | "cat" .
  OptionalCat = "cat"
             = TwoWheeled "fourbyfour"
  Vehicle
  TwoWheeled = "scooter" | "bicycle" .
              = Parents Children .
  Family
  Children
              = Child Children
              = OptionalDad OptionalMom | "Mom" "Dad".
  Parents
  OptionalDad = "Dad"
                        .
  OptionalMom = "Mom"
                            "Margaret" | "Alice" | "Robyn" | "Cathy"
                 "Helen"
  Child
              =
                | "Janet" | "Anne" | "Ntombizodwa" | "Ntombizanele" .
END Home.
```

It should be pretty apparent that the productions for Home and Family cause no problems (no alternatives appear in their right hand sides), nor do the productions for Pets, TwoWheeled and Child (they are not nullable, and the alternatives begin with clearly distinct terminals).

The production for Parents needs closer scrutiny.

```
FIRST(Parents_1) = FIRST(OptionalDad) U FIRST(OptionalMom) = { "Dad", "Mom" }
(because OptionalDad is nullable)
FIRST(Parents 2) = { "Mom" }
```

so Rule 1 is broken, and the grammar is not LL(1) compliant.

We can check Rule 2, as there are several productions that have alternatives, one of which is nullable. These are the productions for AllPets, OptionalCat, Vehicle, Children, Parents, OptionalDad and OptionalMom (yes, Sylvia, real grammars often have lots of exciting complications).

This means that we must look at

FIRST(AllPets) and FOLLOW(AllPets) FIRST(OptionalCat) and FOLLOW(OptionalCat) FIRST(Vehicle) and FOLLOW(Vehicle) etc.

The results follow

```
FIRST(AllPets) = { "dog", "cat" }
FOLLOW(AllPets) = { "house", "scooter", "bike" }
FIRST(OptionalCat) = { "cat" }
FOLLOW(OptionalCat) = { "dog", "cat", "house", "scooter", "bike" }
```

(so Rule 2 is broken here, perhaps surprisingly)

FIRST(Vehicle) FOLLOW(Vehicle)	=	{ {	"scooter", "bicycle" } "house" }
FIRST(Children) FOLLOW(Children)	=	{ {	"Helen", "Margaret", "Alice" "Ntombizanele" } "dog", "cat", "house", "scooter", "bike" }
FIRST(Parents) FOLLOW(Parents)	=	{ {	"Mom", "Dad" } "Helen", "Margaret", "Alice" "Ntombizanele", "dog", "cat", "house", "scooter", "bike" }
FIRST(OptionalDad) FOLLOW(OptionalDad)	=	{ {	"Dad" } "Mom, "Helen", "Margaret", "Alice" "Ntombizanele", "dog", "cat", "house", "scooter", "bike" }
FIRST(OptionalMom) FOLLOW(OptionalMom)	= =	{ {	"Mom" } "Helen", "Margaret", "Alice" "Ntombizanele", "dog", "cat", "house", "scooter", "bike" }

We can get an LL(1) description of the family as follows:

```
Home5 = Family ( Pets ) [ Vehicle ] "house" .

Pets = "dog" | "cat" .

Vehicle = ( "scooter" | "bicycle" ) "fourbyfour" .

Family = Parents ( Child ) .

Parents = [ "Dad" [ "Mom" [ "Dad" ] ] .

Child = "Helen" | "Margaret" | "Alice" | "Robyn" | "Cathy"

| "Janet" | "Anne" | "Ntombizodwa" | "Ntombizanele" .
```

Task 2 - Expressions - again

The following grammar attempts to describe expressions incorporating the familiar operators with their correct precedence and associativity.

```
COMPILER Expression $CNF

IGNORE CHR(O) .. CHR(31)

PRODUCTIONS

Expression = Term { ( "+" | "-" ) Term }.

Term = Factor { ( "*" | "/" ) Factor }.

Factor = Primary [ "↑" Expression ].

Primary = "a" | "b" | "c".

END Expression.
```

Is this an ambiguous grammar? (Hint: try to find an expression that can be parsed in more than one way).

Again, many people "guessed" the right answer. To justify the claim that it is ambiguous it would be as well to

Computer Science 301 - 2010 - Practical 22 solutions

show a pair of parse trees, not just make a wild claim! Considering the expression $a \uparrow b \star c$. This can indeed be parsed in two ways, one with the implicit meaning of $a \uparrow (b \star c)$ and the other with the meaning of $(a \uparrow b) \star c$. The parse trees would look like this (a few intermediate nodes have been omitted to save space)



Is it an LL(1) grammar? If not, why not, and can you find a suitable grammar that is LL(1)?

It cannot be an LL(1) grammar if it is ambiguous, but let us see which rules are broken. If we rewrite the first grammar to eliminate the metabrackets we get

```
Expression = Term TailExp .
TailExp
          = AddOp Term TailExp ε.
           = Factor TailTerm .
Term
          = MulOp Factor TailTerm | ε .
TailTerm
Factor
           = Primary TailFactor .
TailFactor = "\uparrow" Expression | \varepsilon .
         = "a"
Primary
                   "b" | "c"
                              .
           = "+"
                  "-"
add0p
                  | "/" ]
           = "*"
MulOp
```

The nullable nonterminals here are TailExp, TailTerm and TailFactor.

```
FIRST(TailExp) = { "+" , "-" }
FIRST(TailTerm) = { "*" , "/" }
FIRST(TailFactor) = { "^" }
```

The FOLLOW sets are a little harder to see because to get to closure one has to chase through quite a few other productions:

```
FOLLOW(TailExp) = FOLLOW(Expression)
FOLLOW(TailTerm) = FOLLOW(Term) = FIRST(TailExp) U FOLLOW(Expression)
FOLLOW(TailFactor) = FOLLOW(Factor) = FIRST(TailTerm) U FOLLOW(Term)
```

You are invited to track these through in detail; the outcome is that they are all the same:

FOLLOW(TailExp) = { "*" , "/" , "+" , "-" , EOF } FOLLOW(TailTerm) = { "*" , "/" , "+" , "-" , EOF } FOLLOW(TailFactor) = { "*" , "/" , "+" , "-" , EOF }

and so Rule 2 is broken for TailExp and for TailTerm.

Finding an LL(1), unambigous grammar, with the correct precedence and associativity is not too difficult. In fact it would have been incredibly easy had you just read the text, page 127, where the solution is effectively given to you.

```
COMPILER Expression $CNF

IGNORE CHR(0) .. CHR(31)

PRODUCTIONS

Expression = Term { ( "+" | "-" ) Term } .

Term = Factor { ( "*" | "/*" ) Factor } .

Factor = Primary [ "↑" Factor ] .

Primary = "a" | "b" | "c" .

END Expression.
```

Computer Science 301 - 2010 - Practical 22 solutions

Why does our grammar now satisfy the LL(1) constraints? Rewritten it becomes

```
Expression = Term TailExp .

TailExp = AddOp Term TailExp | ε.

Term = Factor TailTerm .

TailTerm = MulOp Factor TailTerm | ε.

Factor = Primary TailFactor .

TailFactor = "↑" Factor | ε.

Primary = "a" | "b" | "c".

AddOp = "+" "-".

MulOp = "*" | "/".
```

The nullable nonterminals here are still TailExp, TailTerm and TailFactor.

FIRST(TailExp) = { "+" , "-" } FIRST(TailTerm) = { "*" , "/" } FIRST(TailFactor) = { "↑" }

The FOLLOW sets are a little harder to see because to get to closure one has to chase through quite a few other productions:

```
FOLLOW(TailExp)= FOLLOW(Expression)FOLLOW(TailTerm)= FOLLOW(Term) = FIRST(TailExp) U FOLLOW(Expression)FOLLOW(TailFactor)= FOLLOW(Factor) = FIRST(TailTerm) U FOLLOW(Term)
```

You are invited to track these through in detail; the outcome is:

```
FOLLOW(TailExp) = { EOF }
FOLLOW(TailTerm) = { "+" , "-" , EOF }
FOLLOW(TailFactor) = { "*" , "/" , "+" , "-" , EOF }
```

and so Rule 2 is no longer broken.

There were various other suggestions made, such as

Factor = Primary ["↑" Term] .
Factor = Primary ["↑" "(" Expression ")"] .
Factor = Primary { "↑" Term } .

but these are unnecessarily restrictive (first suggestion) or non-equivalent (second suggestion; parentheses were not catered for in the first grammar and introducing them is "cheating"). The third suggestion gets the associativity incorrect.

Task 3 - Palindromes

Palindromes are character strings that read the same from either end. You were invited to explore various ways of finding grammars that describe palindromes made only of the letters a and b:

Palindrome = "a" Palindrome "a" | "b" Palindrome "b".
 Palindrome = "a" Palindrome "a" | "b" Palindrome "b" | "a" | "b".
 Palindrome = "a" [Palindrome] "a" | "b" [Palindrome] "b".
 Palindrome = ["a" Palindrome "a" | "b" Palindrome "b" | "a" | "b"].

Which grammars achieve their aim? If they do not, explain why not. Which of them are LL(1)? Can you find other (perhaps better) grammars that describe palindromes and which *are* LL(1)?

This is one of those awful problems that looks deceptively simple, and indeed is deceptive. We need to be able to cater for palindromes of odd or even length, and we need to be able to cater for palindromes of finite length, so that the "repetition" that one immediately thinks of has to be able to terminate.

Here are some that don't work:

```
COMPILER Palindrome /* does not terminate */

PRODUCTIONS

Palindrome = "a" Palindrome "a" | "b" Palindrome "b".

END Palindrome.

COMPILER Palindrome /* only allows odd length palindromes */

PRODUCTIONS

Palindrome = "a" Palindrome "a" | "b" Palindrome "b" | "a" | "b".

END Palindrome.

COMPILER Palindrome /* only allows even length palindromes */

PRODUCTIONS

Palindrome = "a" [ Palindrome ] "a" | "b" [ Palindrome ] "b".

END Palindrome.
```

Of those grammars, the first *seems* to obey the LL(1) rules, but it is useless (it is not "reduced" in the sense of the definitions on page 129). The second one is obviously non-LL(1) as the terminals "a" and "b" can start more than one alternative. The third one is less obviously non-LL(1). If you rewrite it

```
COMPILER Palindrome /* only allows even length palindromes */
PRODUCTIONS
Palindrome = "a" Extra "a" | "b" Extra "b" .
Extra = Palindrome | ε.
END Palindrome.
```

```
and note that Extra is nullable, then FIRST(Extra) = { "a", "b" } and FOLLOW(Extra) = { "a", "b" }.
```

Here is another attempt

```
COMPILER Palindrome /* allows any length palindromes */
PRODUCTIONS
Palindrome = [ "a" Palindrome "a" | "b" Palindrome "b" | "a" | "b" ].
END Palindrome.
```

This describes both odd and even length palindromes, but is non-LL(1). Palindrome is nullable, and both FIRST(Palindrome) and FOLLOW(Palindrome) = $\{ "a", "b" \}$. And, as most were quick to notice, it breaks Rule 1 immediately as well.

Other suggestions were:

```
COMPILER Palindrome /* allows any length palindromes */

PRODUCTIONS

Palindrome = "a" [ Palindrome "a"] | "b" [ Palindrome "b" ].

END Palindrome.
```

but, ingenious as this appears, it does not work either. Rewritten it would become

```
COMPILER Palindrome /* allows any length palindromes */

PRODUCTIONS

Palindrome = "a" PalA | "b" PalB .

PalA = Palindrome "a" | .

PalB = Palindrome "b" | .

END Palindrome.
```

```
PalA and PalB are both nullable, and FIRST(PalA) = { "a" , "b" } while FOLLOW(PalA) =
FOLLOW(Palindrome) = { "a", "b" } as well.
```

In fact, when you think about it, you simply will not be able to find an LL(1) grammar for this language. (That is fine; grammars don't have to be LL(1) to be valid grammars. They just have to be LL(1) or very close to LL(1) to be able to write recursive descent parsers.) Here's how to think about it. Suppose I asked you to hold your breath for as long as you could, and also to nod your head when you were half way through. I don't believe you could do it - you don't know before you begin exactly how long you will be holding your breath. Similarly, if I told you to get into my car and drive it till the tank was empty but to hoot the hooter when you were half way to running out you could not do it. Or if I told you to walk into a forest with your partner and kiss him/her when you were in the dead centre of the forest, you would not know when the magic moment had arrived.

LL(1) parsers have to be able to decide just by looking at one token exactly what to do next - if they have to *guess* when they are are half-way through parsing some structure they will not be able to do so. One would have to stop applying the options like Palindrome = "a" Palindrome "a" at the point where one had generated or analyzed half the palindrome, and if there is no distinctive character in the middle one would not expect the parser to be able to do so.

If course, if one changes the problem ever so slightly in that way one *can* find an LL(1) grammar. Suppose we want a grammar for palindromes that have matching a and b characters on either end and a distinctive c or pair of c characters in the centre:

```
COMPILER Palindrome /* allows any length palindromes, but c must be in the middle */
PRODUCTIONS
Palindrome = "a" Palindrome "a" | "b" Palindrome "b" | "c" [ "c" ] .
END Palindrome.
```

Several submissions suggested (but did not justify) that maybe this problem could be solved by using a contextsensitive set of productions (which would not be LL(1)). That may be possible - I must think about it some more. Context-sensitive grammars are awkward to work with!

Task 4 - Pause for thought

Which of the following statements are true? Justify your answer.

- (a) An LL(1) grammar cannot be ambiguous.
- (b) A non-LL(1) grammar must be ambiguous.
- (c) An ambiguous language cannot be described by an LL(1) grammar.
- (d) It is possible to find an LL(1) grammar to describe any non-ambiguous language.

To answer this sort of question you must be able to argue convincingly, and most people did not do that at all!

(a) is TRUE. An LL(1) grammar cannot be ambiguous. If a language can be described by an LL(1) grammar it will always be able to find a single valid parse tree for any valid sentence, and no parse tree for an invalid sentence. The rules imply that no indecision can exist at any stage - either you can find a unique way to continue the implicit derivation from the goal symbol, or you have to conclude that the sentence is malformed.

But you cannot immediately conclude any of the "opposite" statements, other than (c) which is TRUE. If you *really* want to define an ambiguous language (and you may have perfectly good/nefarious reasons for doing so - stand-up comedians do it all the time) you will not be able to describe it by an LL(1) grammar, which has the property that it can only be used for deterministic parsing.

In particular (b) is FALSE. We can "justify" this by giving just a single counter example to the claim that it might be true. We have seen several such grammars. The palindrome grammars above are like this - even though they are non LL(1) for the reasons given, they are quite unambiguous - you would only be able to parse any palindrome in one way! Many people seem not to realise this - they incorrectly conclude that non-LL(1) inevitably implies ambiguity. The other classic case is that of the left-recursive expression grammars discussed in class and in chapter 6.1.

Similarly, a variation on the train grammar simplified to

Train = "loco" "coal" { "coal" | "fuel" } "coal" "guard" "." .

is non-LL(1), but it is not ambiguous - you could only parse the train

loco coal coal coal guard .

in one way. This is a particularly simple grammar and it is hopefully easy to see that *any* valid train defined by it could only be parsed in one way.

Similarly (d) is FALSE. Once again the palindrome example suffices - this language is simple, unambiguous, but we can easily argue that it is impossible to find an LL(1) grammar to describe it.

Task 6 - Eliminating meta-brackets

As hinted in the problem description, the trick here is to replace the productions that use meta-braces for repetition and meta-brackets for optional selection by ones that use right recursion (to avoid LL(1) violations). Here is one possibility:

```
COMPILER EBNF1 $CN
/* Recognize a set of EBNF productions
   (Does not permit empty terms)
   P.D. Terry, Rhodes University, 2010 */
CHARACTERS
 control = CHR(0) .. CHR(31) . // '\u0000' .. '\u001f' . // Linz version
           = "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz" .
  letter
          = " " .
  lowline
           = "0123456789" .
 digit
 noQuote1 = ANY - "'" - control .
 noQuote2 = ANY - '"' - control .
TOKENS
 nonTerminal = letter { letter | lowline | digit } .
terminal = "'" noQuote1 { noQuote1 } "'" | '"' noQuote2 { noQuote2 } '"' .
COMMENTS FROM "(*" TO "*)" NESTED
IGNORE control
PRODUCTIONS
 EBNF1
              = Productions EOF .
 Productions = Production Productions .
 Production = nonTerminal "=" Expression ".".
 Expression = Term MoreTerms
             = "|" Term MoreTerms | .
 MoreTerms
              = Factor MoreFactors
 Term
 MoreFactors = Factor MoreFactors
                 nonTerminal terminal
  Factor
                | "[" Expression "]" | "(" Expression ")" | "{" Expression "}" .
END EBNF1.
```

This can be further rearranged to give an even more compact grammar (you might like to ponder whether it shows any differences from the original one so far as associativity and precedence are concerned).

```
PRODUCTIONS

EDNF2 = Productions EOF.

Productions = Production Productions | .

Production = nonTerminal "=" Expression ".".

Expression = Term MoreTerms.

MoreTerms = "|" Expression | .

Term = Factor MoreFactors.

MoreFactors = Term | .

Factor = nonTerminal | terminal

| "[" Expression "]" | "(" Expression ")" | "{" Expression "}".

END EBNF2.
```

The above grammars match the ones given in the problem description. They do not, however, have the property of being able to describe themselves any longer - we might argue that we need to be able to describe a nullable factor (the original could not do this). This might be achieved as follows:

```
PRODUCTIONS

EBNF3 = Productions EOF.

Productions = Production Productions | .

Production = nonTerminal "=" Expression "." .

Expression = Term MoreTerms .

MoreTerms = "|" Expression | .

Term = Factor Term | .

Factor = nonTerminal | terminal

| "[" Expression "]" | "(" Expression ")" | "{" Expression "}" .

END EBNF3.
```

but notice that this also allows you to accept production rules like

```
A = b | c | | .
```

which you might argue is a bit silly. Further reflection on this is left as a useful exercise!

Task 7 - Describe BNF

The development of an EBNF description of BNF is shown below. There are a few tricks to be learned from this one. Firstly, productions are separated one from the next by the end of line, not by a period. This means that we cannot IGNORE the line break characters. This has to be done in a way that depends on your operating system, in general. In practice, we have little alternative but to define the eol "character set" as the singleton CHR (10) and then define an EOL "token" as a single character token, as in the code below. Internally Coco/R has been arranged to map line breaks demarcated by lf (CHR(10)) as in Unix, or by cr + lf (as in WinTel) or even cr only (as on Apple computers) to a consistent lf. Secondly, we wish spaces to become significant characters within the nonTerminal tokens that are demarcated by < > brackets. Thirdly, we do *not* want to define the terminal token to include spaces within it, as we need to be able to distinguish each terminal from the next if and when they are separated by spaces. In Cocol there is an implicit IGNORE CHR(32) - but this relates to ignoring spaces *between* tokens, as is common in almost all programming languages. The only way we can make spaces significant *within* a token definition is to define the singleton character set space to consist of CHR(32), as Coco also forbids you from writing a string into a Cocol definition with an embedded spaces, as exemplified by "this has some spaces". Lastly, BNF notation still allows for the use of (round) parentheses (it is tempting to discard these as well).

Incidentally, spaces are very rarely significant in computer languages - the definition of nonTerminal here is one of the very few exceptions one can think of (other than the obvious "string with spaces").

A simple definition of the possible tokens looks like this:

```
CHARACTERS

letter = "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz" .

alpha = letter + "0123456789_" .

lf = CHR(10) .

space = CHR(32) .

TOKENS

EOL = lf .

nonTerminal = "<" { space } letter { alpha | space } ">" .

terminal = letter { alpha } .
```

However, this is not really adequate. We should be able to allow almost anything as a "terminal". But a definition like this is doomed to failure:

```
CHARACTERS
              = CHR(10)
  l f
 space
              = CHR(32).
              = "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz" .
  letter
 alpha
              = letter + "0123456789 "
              = ANY - CHR(0) CHR(\overline{3}2)
  inTerm
TOKENS
 EOL
              = lf .
 nonTerminal = "<" { space } letter { alpha | space } ">" .
              = inTerm { inTerm } .
 terminal
```

because one cannot then distinguish terminals from non-terminals (why not?). We might try

```
CHARACTERS
  control
              = CHR(0) .. CHR(31) .
  letter
              = "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz" .
              = letter + "0123456789_" .
 alpha
              = ANY - control - " "
 graphic
              = graphic - "<" .
 startTerm
              = CHR(10) .
  lf
              = CHR(32).
 space
TOKENS
              = | f
 EOL
 nonTerminal = "<" { space } letter { alpha | space } ">" .
              = startTerm { graphic } .
  terminal
```

but this is also inadequate, as a sequence like (oneterm|twoTerm) with no helpful spaces will all be scanned as a single terminal, and furthermore there is no way to represent the metasymbol < as a terminal in a set of

productions (or any of the other metasymbols, for that matter). Perhaps the merits of the Wirth/Cocol notation are now becoming more apparent! One could, of course, try to insist that users insert space around all terminals, but to be more helpful it may be best to exclude all the meta-characters from starting a terminal, and then to insist that if one wants them as terminals one should use a 'string' notation after all. So a complete grammar might reads like this, although this allows one to have multiple and potentially misleading *eps* in a term, as in

```
<A> ::= a eps eps eps b
COMPILER BNF1 $CN
/* Grammar to describe BNF productions
  P.D. Terry, Rhodes University, 2007 */
CHARACTERS
             = CHR(0) ... CHR(31).
 control
             = "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz" .
 letter
             = letter + "0123456789_" .
 alpha
 graphic
             = ANY - control - " "
                                   .
            = graphic - "'".
= graphic - "()<|:'".
 noQuote
 startTerm
 lf
             = CHR(10).
             = CHR(32).
 space
TOKENS
 EOL
             = lf .
 nonTerminal = "<" { space } letter { alpha | space } ">" .
             = startTerm { graphic } | "'" noQuote { noQuote | "''" } "'" .
 terminal
COMMENTS FROM "(*" TO "*)" NESTED
COMMENTS FROM "/*" TO "*/" NESTED
IGNORE control - lf
PRODUCTIONS
            = { Production } EOF .
 BNF1
 Production = nonTerminal "::=" Expression SYNC EOL .
 Expression = Term { " | Term } .
          = Factor { Factor } .
 Term
             = nonTerminal | terminal | "(" Expression ")" | "eps" .
 Factor
END BNF1.
```

A slightly better solution is to have the PRODUCTIONS section reading:

```
PRODUCTIONS

BNF2 = { Production } EOF .

Production = nonTerminal "::=" Expression SYNC EOL .

Expression = Term { " | " Term } .

Term = Factor { Factor } | "eps" .

Factor = nonTerminal | terminal | "(" Expression ")" .

END BNF2.
```

although that still allows one to have multiple and potentially misleading

| eps | eps | eps

options in an *Expression*. If one want to restrict the right hand side to contain at most one *eps*, and still to have an LL(1) grammar, one is forced to demand that the *eps* appears first, as in the grammar below:

```
PRODUCTIONS
BNF3 = { EOL } { Production } EOF .
Production = nonTerminal "::=" Expression EOL { EOL }.
Expression = [ "eps" "|" ] Term { "|" Term } .
Term = Factor { Factor } .
Factor = nonTerminal | terminal | "(" Expression ")" .
END BNF.
```

Notice that this last grammar also allows you to have blank lines between productions, which the others do not - do you see why?

Task 8 - The Railways are looking to employ skilled programmers

The suggested grammar fails on two scores - it is not LL(1), and it requires at least two safe trucks in every train.

```
PRODUCTIONS

Train2 = { OneTrain } EOF .

OneTrain = LocoPart [ Passengers | FreightOrMixed ] SYNC "." .

LocoPart = "loco" { "loco" } .

FreightOrMixed = SafeTruck { AnyTruck } LastPart .

LastPart = "brake" | SafeTruck Passengers .

Passengers = { "coach" } "guard" .

SafeTruck = "closed" | "coal" | "open" | "cattle" .

AnyTruck = SafeTruck | "fuel" .

END Train2.
```

Why is it not LL(1) compliant? We could apply all the theory of Chapter 7 of the textbook, but maybe an example will suffice. Suppose we have a valid train like

loco coal coal coal coal coach guard

The first coal truck is parsed by the leading SafeTruck in GoodsPart. The next two coal trucks must be parsed by the repetitive part { AnyTruck }, but you can probably see that the last coal truck would have to be parsed by the alternative within LastPart. Unfortunately an LL(1) parser can't see far enough ahead to make that decision, and would be tempted to treat this last coal truck as part of the { AnyTruck } sequence.

It is remarkable that some problems that at first sight look so simple often turns out to be frustratingly difficult. Not being able to find an LL(1) grammar is not a train smash - one quite often cannot find an LL(1) grammar for a language. But it's usually worth a try, as parsers for LL(1) grammars are so easy to write. The clue is to be found in a suggestion that one should factorize the grammar not to concentrate on the "obvious" types of train, but on the requirement that at any point along the train the remainder of the train should be "safe". Thus:

```
PRODUCTIONS
 Train4
                 = { OneTrain } EOF .
                 = LocoPart [ SafeLoad | "brake" | Passengers ] SYNC "." .
 OneTrain
 LocoPart
                 = "loco" { "loco" } .
                 = { "coach" } "guard"
 Passengers
 SafeLoad
                 = SafeTruck { SafeTruck } ( "brake" | Passengers | SafeFuel ) .
                 = "fuel" { "fuel" } ( SafeLoad | "brake" ) .
 SafeFuel
                 = "coal" | "closed" | "open" | "cattle" .
 SafeTruck
END Train4.
```

You might be interested to look at two other solutions submitted by students. How, if at all, are these different? Are they correct? Are they LL(1) grammars?

```
PRODUCTIONS // Lee Stack and Iain Davis, 2010
Trains = { OneTrain } EOF .
OneTrain = LocoPart [ Passengers | SafeTruck FreightOrMixed | "brake"] SYNC "." .
LocoPart = "loco" { "loco" } .
FreightOrMixed = { "fuel" } (SafeTruck FreightOrMixed | "brake" ) | Passengers.
Passengers = { "coach" } "guard".
SafeTruck = "coal" | "closed" | "open" | "cattle" .
END Trains.
```

This last one has had the metabrackets removed. Names like A, B, C etc are little unhelpful, however.

```
PRODUCTIONS // Rex, Zengeni, Thorne, 2010
 Trains
                 = A EOF .
                 = OneTrain A
 А
                 = LocoPart B SYNC " "
 OneTrain
                 = Passengers | FreightOrMixed | "brake" | "guard" | .
 в
                 = "loco" C
 LocoPart
                 = "loco" C | .
 С
 FreightOrMixed = SafeTruck G D.
                = SafeTruck G
 G
                 = Truck | LastPart
= "brake" | Passengers
 D
 LastPart
                 = "coach" E "guard" .
 Passengers
                 = "coach" E
                 = "coal" | "closed" | "open" | "cattle" .
 Safe⊺ruck
                 = "fuel" F (FreightOrMixed | "brake").
 Truck
                 = "fuel" F | .
END Trains.
```